Phong Pham
Sect 202

32/30

STT 200 Section 202 8am Midterm Exam

Unless requested, do not evaluate answers.

1. A random variable X has the following probability distribution:

x	0	4	5
p(x)	0.7	0.1	0.2

a. E X $= 0(.7) + 4(.1) + 5(.2) = 1.4$

b. E X^2 $= 0^2(.7) + 4^2(.1) + 5^2(.2) = 6.6$

c. Var X $= EX^2 - (EX)^2 = 6.6 - (1.4)^2 = 4.64$

d. Standard deviation of X

$\sigma = \sqrt{4.64} = 2.154$

2. A random variable X has E X = 2, variance X = 9. Denote by T the random **total** of 100 independent plays of X.

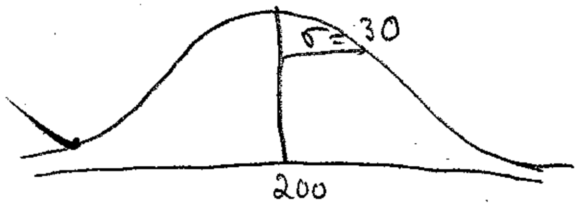
a. E T $= 100\,EX = 100(2) = 200$

b. Var T $= 100 \cdot \mathrm{Var}\,X = 100(9) = 900$

c. Standard deviation of T

$\sqrt{900} = 30$

d. Sketch the approximate distribution of total T, clearly indicating the means and standard deviation of T as recognizable elements in your sketch.



e. Determine the standard score z of total = 245.

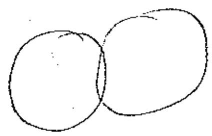
$z = \frac{245-200}{30} = 1.5.$

3. Events A, B have

P(A) = 0.2

P(B | IF A) = 0.6

P(B | IF A^C) = 0.3



~~P(B) = .8~~
~~P(B).~~

a. P(A and B) = P(A) · P(B |if A) = .2(.6) = .12

b. P(A^C and B) = P(A^c) · P(B | if A^c) = .8 (.3) = .24

c. P(B) = P(A and B or A^c and B) = .12 + .24 = .36

d. P(A | IF B) = $\frac{P(\text{A and B})}{P(B)} = \frac{.12}{.36} = .3\bar{3}$

P(B and A) =

$Var(ax) = a^2 Var x$

$Var(-bY) = (-b)^2 Var(Y)$

4. In terms of constants a, b, c and Var X, Var Y of independent random variables X, Y,

a. E (a X − b Y + c) $= a EX - bEY + C$

b. Var(a X − b Y + $\cancel{c}$) $= a^2 Var(X) + b^2 Var(Y)$

5. Box 1 has 8R 2G
Box 2 has 3R 7G
Box 1 will be chosen with probability 0.4
Box 2 will be chosen with probability 0.6

A ball will be selected with equal probability from the box chosen.

a. Intuitively, is P(Box 1 |IF R) or P(Box 1) the larger? Why? In general we will say P(Box1) because, P(Box1|ifR) is hardly to get, because R is in 2 boxes. The percent to get Box1 is higher. It is slightly 50:50 chance. But in this problem we set 0.4. After I calculate P(Box1|if R) is bigger in this problem.

b. P(Box 1 and R)

$P(\text{Box 1 and R}) = P(\text{box1}) \cdot P(\text{R if Box1}) = .4(8/10) = .32$

c. P(R) $= P(\text{Box1 and R or box 2 and R}) =$

$\Rightarrow P(\text{Box1 and R}) + P(\text{box2 and R}) = P(\text{Box1 and R}) + P(\text{box 2}) P(\text{R if box 2})$

$= .32 + .6(3/10) = \boxed{.5}$

d. P(Box 1 |IF R). $= \dfrac{P(\text{Box1 and R})}{P(R)} = \dfrac{.32}{.5} = .64$

6. Calculator may be used.

$$\begin{pmatrix} x & (x-\text{Mean}[x])^2 & x^2 \\ 1 & 25 & 1 \\ 2 & 16 & 4 \\ 6 & 0 & 36 \\ 15 & 81 & 225 \\ \_ & \_ & \_ \\ 24 & 122 & 266 \end{pmatrix}$$

(totals at bottom)

$$\sigma^2 = \frac{\sum (x-\mu)^2}{n}$$

a. Standard deviation σ of list x.

$$\sigma^2 = \frac{\sum (x-\mu)^2}{n} = \frac{122}{4} = 30.5 \to \sigma = \sqrt{30.5} = 5.523$$

b. Median of list x.

$$\frac{2+6}{2} = 4.$$

c. Standard deviation σ of list 3x - 4.

$$\sigma_{3x-\cancel{4}} = |3|\sigma_x = 3(5.523) = 16.568$$

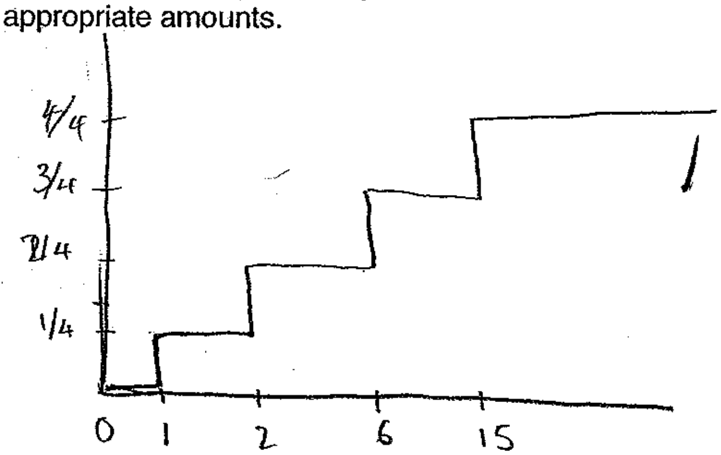
d. Median of list 3x - 4. $= 3\cdot(\text{median } x) - 4$

$$= 3(4) - 4$$
$$= 8$$

e. Height of the probability histogram for list x over the interval [1.5, 13.5].

$$\Rightarrow \frac{(2/4)}{13.5 - 1.5} = .041\overline{6}$$

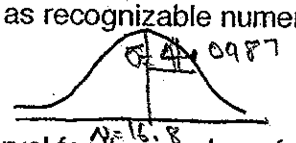
f. Sketch the cumulative probability distribution for list x. Be sure it rises between 0 and 1 with jumps of appropriate amounts.



$\mu = 16.8$

7. Poisson. We expect on average 16.8 electrical outages per storm and the number of outages is thought to follow a Poisson distribution. Recall that for the Poisson the mean and variance are the same.

a. Sketch the approximate distribution of the Poisson for this case. Label the numerical mean and standard deviation as recognizable numerical elements of your sketch.



$\mu = 16.8$

$\sigma = \sqrt{\mu} = \sqrt{16.8} = 4.0987$

b. Give a 95% interval for the number of electrical outages in a given storm.

$\mu \pm 1.96\sigma$

$X_{low} = 16.8 - 1.96 \cdot (4.0987) = 8.766$

$X_{high} = 16.8 + 1.96(4.0987) = 24.833$

c. The formula for Poisson p(x) is $e^{-\mu}\frac{\mu^x}{x!}$ for x = 0, 1, ...

$$p(15) = \frac{e^{-16.8}(16.8)^{15}}{15!} = \frac{e^{-16.8}(16.8)^{15}}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdots (1)} = .09268$$

(first write all appropriate numbers in the formula, evaluate the factorial, then evaluate using calculator).

$15! = 1.307 \times 10^{12}$

8. Binomial. Each time a purchase is made at a vending machine there is probability 0.4 that change will be required. These events are thought to be pretty much independent.

C N

a. The probability that the first four purchases are:
change no-change no-change change

C N N C

$= .4(.6)(.6)(.4) = .0576$

b. The number of ways to select two of four positions as the only ones requiring change.

$$C(4,2) = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2}{2 \cdot 2} = 6$$

c. The binomial probability that out of four purchases there are precisely two that require change.

$$C(4,2)(.0576)^6 = \frac{4!}{2!2!}(.0576) = .3456$$

d. Sketch the approximate distribution of the number X of purchases, out of 100 purchases, that require change. Identify the mean and standard deviation of X numerically in your sketch.

$\mu = np = 100(.4) = 40$

$\sigma = \sqrt{np(1-p)} = \sqrt{100(.4)(.6)} = 4.89$